Find the following for the vectors $\vec{u} = 7i - 5j + \sqrt{7}k$ and $\vec{v} = -7i + 5j - \sqrt{7}k$.

a. $\vec{v} \cdot \vec{u}$, $|\vec{v}|$, and $|\vec{u}|$

b. the cosine of the angle between $\vec{v}$ and $\vec{u}$.

c. the scalar component of $\vec{u}$ in the direction of $\vec{v}$

d. the vector $\text{proj}_{\vec{v}}\vec{u}$

$$\vec{v} = -7i + 5j - \sqrt{7}k. \qquad \vec{u} = 7i - 5j + \sqrt{7}k$$

a. $\vec{v} \cdot \vec{u} = (-7)(7) + (5)(-5) + (-\sqrt{7})(\sqrt{7})$
$= -49 - 25 - 7$
$= -81$

dot product

$|\vec{v}| = \sqrt{(-7)^2 + 5^2 + (-\sqrt{7})^2}$ $\quad |\vec{v}| = \sqrt{v_1^2 + v_2^2}$ magnitude $\quad |\vec{u}| = \sqrt{(7)^2 + (-5)^2 + (\sqrt{7})^2}$

$= \sqrt{49 + 25 + 7} \qquad = \sqrt{49 + 25 + 7}$

$= \sqrt{81} = 9 \qquad = \sqrt{81} = 9$

b. $\cos\theta^{-1} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$ $\quad \cos\theta^{-1}$ " inverse cosine"

cosine of angle

$= \left(\dfrac{-81}{9 \cdot 9}\right)$

$= -1$

c. $|\vec{u}| \cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$

scalar component

$= \dfrac{-81}{9}$ $\quad$ or $9(-1) = -9$

$= -9$

d. $\text{Proj}_{\vec{v}} |\vec{u}| = (|\vec{u}| \cos\theta) \dfrac{\vec{v}}{|\vec{v}|}$

vector projection of $\vec{u}$ onto $\vec{v}$

$= \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v}$ $\quad$ magnitude × direction !!!

$= \left(\dfrac{-81}{9^2}\right)(-7i + 5j - \sqrt{7}k)$

$= \left(\dfrac{-81}{81}\right)(-7i + 5j - \sqrt{7}k)$

$= (-1)(-7i + 5j - \sqrt{7}k)$

$= 7i - 5j + \sqrt{7}k$

Given the vectors $\vec{v}$ and $\vec{u}$: $\vec{v} = 11i + 2j - 10k$ $\quad \vec{u} = 8i + 15j$

a. Dot Product

$$\vec{u} \cdot \vec{v} = (8)(11) + (15)(2) + (0)(-10)$$
$$= 88 + 30$$
$$= 118$$

magnitude

$$|\vec{v}| = \sqrt{11^2 + 2^2 + (-10)^2} \qquad |\vec{v}| = \sqrt{v_1^2 + v_2^2}$$
$$= \sqrt{121 + 4 + 100}$$
$$= \sqrt{225}$$
$$|\vec{v}| = 15$$

$$|\vec{u}| = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64 + 225}$$
$$= \sqrt{289}$$
$$|\vec{u}| = 17$$

b. Cosine of the angle between $\vec{v}$ and $\vec{u}$

$$\cos\theta = \frac{118}{17 \cdot 15} \qquad \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$$
$$= \frac{118}{255}$$

c. Scalar component of $\vec{u}$ onto $\vec{v}$

$$|\vec{u}| \cos\theta = \frac{118}{15} \qquad \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$$

d. Vector projection of $\hat{u}$ onto $\hat{v}$.

$$\text{Proj}_{\hat{v}} \hat{u} = \left(\frac{118}{15^2}\right) 11i + 2j - 10k \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v}$$
$$= \frac{1298}{225} i + \frac{236}{225} j - \frac{236}{45} k$$

$\vec{v} = 11i + 2j - 10k$ $\quad \vec{u} = 8i + 15j$

$$\vec{u} \cdot \vec{v} = (8)(11) + (15)(2) + (0)(-10)$$
$$= 88 + 30$$
$$= 118$$

$$|\vec{v}| = \sqrt{11^2 + 2^2 + (-10)^2}$$
$$= \sqrt{121 + 4 + 100}$$
$$= \sqrt{225}$$
$$|\vec{v}| = 15$$

$$|\vec{u}| = \sqrt{8^2 + 15^2}$$
$$= \sqrt{64 + 225}$$
$$= \sqrt{289}$$
$$|\vec{u}| = 17$$

$$\cos\theta = \frac{118}{17 \cdot 15}$$
$$= \frac{118}{255}$$

$$|\vec{u}| \cos\theta = \frac{118}{15}$$

$$\text{Proj}_{\hat{v}} \hat{u} = \left(\frac{118}{15^2}\right) 11i + 2j - 10k$$
$$= \frac{1298}{225} i + \frac{236}{225} j - \frac{236}{45} k$$

Find the angle between the vectors $\vec{u} = 4i - 6j$ and $\vec{v} = -5i + 3j + 5k$

$\theta = \cos^{-1} \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} \rightarrow \cos^{-1}\left[\dfrac{(4)(-5) + (-6)(3) + (0)(5)}{\sqrt{4^2+(-6)^2} \cdot \sqrt{(-5)^2+3^2+5^2}}\right]$ Magnitude $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$\cos^{-1}\left[\dfrac{-38}{\sqrt{52}\cdot\sqrt{59}}\right]$

The angle between the vectors is $\theta \rightarrow \approx$ 2.37 radians

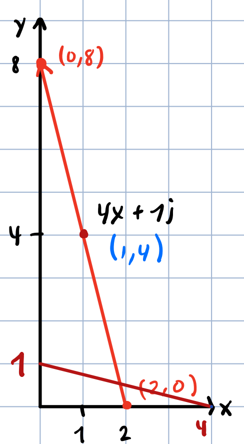
The vector $\vec{v} = ai + bj$ is perpendicular to the line $ax + by = c$. Use this fact to find an equation for the line throug P perpendicular to $\vec{v}$. Then draw a sketch of the line, including $\vec{v}$ as a vector starting at the origin.

$P(1,4)$, $\vec{v} = \overset{x}{4i} + \overset{y}{j}$

substitute $\vec{v} = 4i + j$ in $\overset{i}{a}x + \overset{j}{b}y = c \rightarrow 4x + 1y = 8$

From $P(1,4) = \rightarrow 4(1) + 1(4) = C$

$4 + 4 = 8$



if $ax + by = 4i + j$

then $x=4$ $y=1$ $(4,1)$

set $x = 0$ to get y-intercept

$4x + y = 8 \rightarrow 4(0) + y = 8 \rightarrow y = 8$ $(0,8)$

set $y = 0$ to get x-intercept

$4x + y(0) = 8 \rightarrow 4x = 8 \rightarrow x = 2$ $(2,0)$

The vector $\vec{v} = ai + bj$ is parallel to the line $bx - ay = c$.
Use this fact to find an equation of the line through P parallel to $\vec{v}$. $P(-3,3)$ $\vec{v} = 2i - 3j$

$y - y_0 = m(x - x_0)$
For $\vec{v} = ai + bj$
slope of $\vec{v} = m_1 = \frac{b}{a}$
$bx - ay = c$

substitute $\vec{v} = 2i - 3j$ in $\overset{j}{b}x - \overset{i}{a}y = c \rightarrow 3x + 2y = -3$ $a = 2$ $b = 3$

$bx - ay = c$

$\dfrac{bx - ay}{a} = \dfrac{c}{a}$

$\dfrac{b}{a}x - y = \dfrac{c}{a}$

$y = \dfrac{b}{a}x - \dfrac{c}{a}$

From $P(-3,3) = \rightarrow 3(-3) + (2)(3) = C$

$-9 + 6 = -3$

if $bx - ay = 2i - 3j$

then $x = 2$ $y = -3$ $(2,3)$

$\boxed{\begin{matrix}35-38 \\ 39-42\end{matrix}}$

$-3x - 2y = C$ $\quad -3x - 2 = 3$

$P(-3,3) \rightarrow -3(-3) - 2(3) = C$ $\quad 3x + 2 = -3$

$9 - 6 = 3$

The vector $\vec{v} = ai + bj$ is parallel to the line $bx - ay = c$.
Use this fact to find an equation of the line through P parallel to $\vec{v}$. $P(-2,2)$ $\vec{v} = i - 3j$

substitute $\vec{v} = i - 3j$ in $bx - ay = c$.

$= \; -3x - y = c$

$P(-2,2)$ $\; -3(-2) - 1(2) = c$

$6 - 2 = 4$

$-3x - y = 4$ factor -1

$3x + y = -4$

$x = 0 \rightarrow (0,-4)$

$3(0) + y = -4$

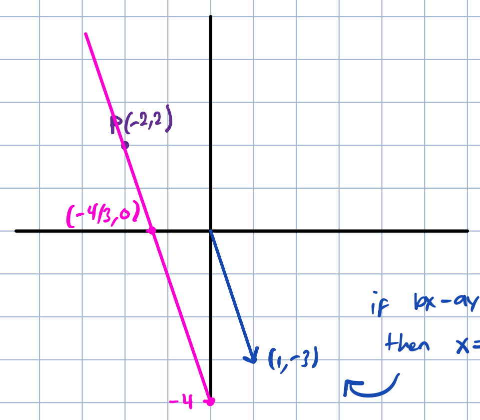
$y = -4$

$y = 0 \rightarrow (-4/3, 0)$ -1.33

$3x + 0 = -4$

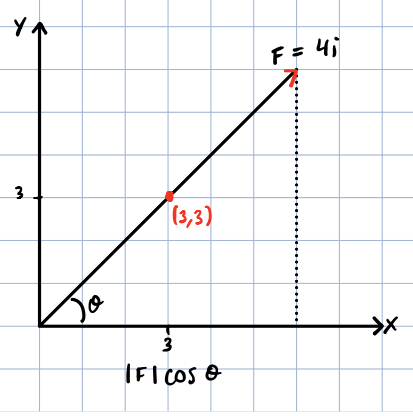
$3x = -4$

$x = -4/3$



if $bx - ay = i - 3j$
then $x = 1$, $y = -3$

Find the work done by a force $F = 4i$ (magnitude 4N) in moving an object along the line from the origin to the point $(3,3)$ (distance in meters)



the distance moved by object is $\vec{D} = \langle 3,3 \rangle$

$|\vec{D}| = \sqrt{3^2 + 3^2}$

$= \sqrt{18} = 3\sqrt{2}$

$\theta = 45°$ since from origin to $(3,3)$

work $= F \cdot D$

$= |F| \cdot |D| \cos\theta$

Trigonometric Ratios Table

cuemath THE MATH EXPERT

θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

$\therefore \; W = 4 \cdot 3\sqrt{2} \cdot \cos(45°)$

$= 4 \cdot 3\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}}$

$= 12J$

How much work does it take to slide a crate 27m along a loading dock by pulling on it with a 220 - N force at an angle of 27° from the horizontal?

$W = |\vec{F}|\, d \cos\theta$

$W = 220 \cdot 27 \cdot \cos 27°$

$= 5940 \cdot \cos 27°$

$= 5292.57 \text{ N}\cdot\text{m} \approx 5293 \text{J}$

The acute angle between intersecting lines that do not cross at right angles is the same as the angle determined by vectors normal to the lines or vectors parallel to the lines.

Use this fact and knowledge of orthogonal and parallel vectors to find the acute angle between the lines $2x - 5y = 5$ and $x - y = 2$.

$ax + by = c$ is perpendicular to $\vec{v} = a\hat{i} + b\hat{j}$

since $2x - 5y = 5$ perpendicular to a vector is $\vec{u} = 2\hat{i} - 5\hat{y}$

$x - y = 2$ perpendicular to a vector is $\vec{v} = 1\hat{i} - 1\hat{j}$

① Find dot product $\quad \vec{u}\cdot\vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2$

$\vec{u}\cdot\vec{v} = \langle 2, -5\rangle + \langle 1, -1\rangle$

$= 2 + 5$

$= 7$

② Find magnitude $|\vec{u}|$ and $|\vec{v}|$ $\quad |\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$|\vec{u}| = \sqrt{2^2 + (-5)^2}$ $\quad |\vec{v}| = \sqrt{(1)^2 + (-1)^2}$

$= \sqrt{29}$ $\quad = \sqrt{2}$

③ Find the angle between the vectors $\quad \theta = \cos^{-1}\left(\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$

$\theta = \cos^{-1}\left(\dfrac{7}{\sqrt{29}\cdot\sqrt{2}}\right)$

$= \cos^{-1}\left(\dfrac{7}{\sqrt{58}}\right) \approx 0.40$ radians

12.4

Find the length and direction (when defined) of $\vec{u} = 6i - 2j - 7k$, $\vec{v} = 7i - 7k$

① $\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ 6 & -2 & -7 \\ 7 & 0 & -7 \end{vmatrix}$ $\quad \vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{vmatrix}$

No j

② $= i \begin{array}{c} j \quad k \\ \begin{vmatrix} -2 & -7 \\ 0 & -7 \end{vmatrix} \end{array} - j \begin{array}{c} i \quad k \\ \begin{vmatrix} 6 & -7 \\ 7 & -7 \end{vmatrix} \end{array} + k \begin{array}{c} i \quad j \\ \begin{vmatrix} 6 & -2 \\ 7 & 0 \end{vmatrix} \end{array}$

why is this positive?

$= i(14-0) - j(-42-(-49)) + k(0-(-14))$

$= i(14) - j(7) + k(14)$

$= 14i - 7j + 14k$

③ Length of $\vec{u}\times\vec{v} = |\vec{u}\times\vec{v}|$

$$= \sqrt{14^2+(-7)^2+(14)^2}$$

$$= \sqrt{441} = \boxed{21}$$

④ Direction of $\vec{u}\times\vec{v} = \dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}$

$$= \frac{14i - 7j + 14k}{21}$$

$$= \boxed{\frac{14i}{21} - \frac{7j}{21} + \frac{14k}{21}}$$

⑤ length of $\vec{v}\times\vec{u} = -(\vec{u}\times\vec{v})$

$$\vec{v}\times\vec{u} = -(14i - 7j + 14k)$$

length of $\vec{v}\times\vec{u}$ = length of $\vec{u}\times\vec{v}$ $= \boxed{21}$

⑥ Direction of $\vec{v}\times\vec{u} = -[\text{direction of } \vec{u}\times\vec{v}]$

$$= \boxed{-\frac{14i}{21} + \frac{7j}{21} - \frac{14k}{21}}$$

Find the length and direction (when defined) of $\vec{u}\times\vec{v}$ and $\vec{v}\times\vec{u}$.

$\vec{u} = -8i + 16j - 20k$, $\vec{v} = -2i + 4j - 5k$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ -8 & 16 & -20 \\ -2 & 4 & -5 \end{vmatrix} = i\begin{vmatrix} 16 & -20 \\ 4 & -5 \end{vmatrix} - j\begin{vmatrix} -8 & -20 \\ -2 & -5 \end{vmatrix} + k\begin{vmatrix} -8 & 16 \\ -2 & 4 \end{vmatrix}$$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{vmatrix}$$

$$= i(\cancel{-80-(-80)}) - j(\cancel{40-40}) + k(\cancel{-32-(-32)})$$

$$= \boxed{0}$$

Length $|\vec{u}\times\vec{v}| = \sqrt{0^2}$

$$= \boxed{0}$$

Direction $\vec{u}\times\vec{v} = \dfrac{0}{0} = \boxed{0}$ the direction is not defined.

$$\frac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}$$

Length $|\vec{v}\times\vec{u}| = -(\vec{u}\times\vec{v})$

$$= \boxed{-(0)}$$

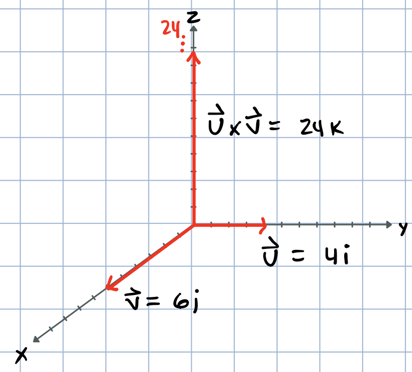
Direction $\vec{v}\times\vec{u} = -[\text{direction of } \vec{u}\times\vec{v}]$

$$= \boxed{-0}$$

the direction is not defined.

Sketch the coordinate axes and then include the vectors of u, v and $u \times v$ as vectors starting from the origin.

$\vec{u} = 4i \qquad \vec{v} = 6j$



Find $\vec{u}\times\vec{v}$ $\rightarrow$ $\begin{vmatrix} i & j & k \\ 4 & 0 & 0 \\ 0 & 6 & 0 \end{vmatrix} = i\begin{vmatrix} 0 & 0 \\ 6 & 0 \end{vmatrix} - j\begin{vmatrix} 4 & 0 \\ 0 & 0 \end{vmatrix} + k\begin{vmatrix} 4 & 0 \\ 0 & 6 \end{vmatrix}$

$= i(0-0) - j(0-0) + k(24-0)$

$= 24k$

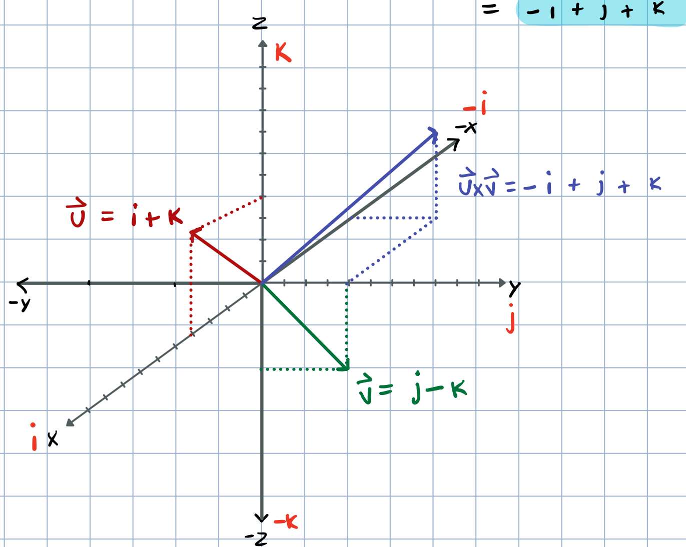
Sketch the coordinate axes and then include the vectors of u, v and $u \times v$ as vectors starting from the origin.

$\vec{u} = i + k \qquad \vec{v} = j - k$

Find $\vec{u}\times\vec{v}$ $\rightarrow$ $\begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = i\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} + k\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$= i(0-1) - j(-1-0) + k(1-0)$

$= -i + j + k$



Find the area of the triangle determined by the points P, Q and R. Find a unit vector perpendicular to plane PQR.

$P(-1, 1, 2), \quad Q(2, 0, 1), \quad R(0, 2, -1)$

$\overrightarrow{PQ} = \langle 2+1, 0-1, 1-2 \rangle$ $\quad \overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= 3i - j - k$

$\overrightarrow{PR} = \langle 0+1, 2-1, -1-2 \rangle$

$= i + j - 3k$

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 3 & -1 & -1 \\ 1 & 1 & -3 \end{vmatrix} = i\begin{vmatrix} -1 & -1 \\ 1 & -3 \end{vmatrix} - j\begin{vmatrix} 3 & -1 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix}$

$= i(3+1) - j(-9+1) + k(3+1)$

$= 4i + 8j + 4k$

① $\text{Area} = \dfrac{\sqrt{4^2+8^2+4^2}}{2} = \dfrac{\sqrt{96}}{2} = \dfrac{\cancel{2}\sqrt{24}}{\cancel{2}} = \sqrt{24}$ or $2\sqrt{6}$ Square units

$\text{Area} = \frac{1}{2}|\vec{PQ} \times \vec{PR}|$

② Unit vector perpendicular to plane PQR.

$$\frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} = \frac{4i+8j+4k}{2\sqrt{6}} = \frac{\cancel{2}(\cancel{2}i+\cancel{4}j+\cancel{2}k)}{\cancel{2}\sqrt{6}} = \frac{1}{\sqrt{6}}(i+2j+k)$$

Find the volume of the parallelepiped (box) determined by $\vec{u}, \vec{v}$ and $\vec{w}$.

$\vec{u}$	$\vec{v}$	$\vec{w}$
$4i+3j$	$4i-j+k$	$4i+3k$

verify that $(\vec{u}\times\vec{v})\cdot\vec{w} = (\vec{v}\times\vec{w})\cdot\vec{u} = (\vec{u}\times\vec{w})\cdot\vec{v}$

triple scalar product as a determinant $(\vec{u}\times\vec{v})\cdot\vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$$(\vec{u}\times\vec{v})\cdot\vec{w} = \begin{vmatrix} 4 & 3 & 0 \\ 4 & -1 & 1 \\ 4 & 0 & 3 \end{vmatrix}$$

$$= 4\begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix} - 0\begin{vmatrix} 4 & 0 \\ 4 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 3 \\ 4 & -1 \end{vmatrix}$$

$$= 4(3+0) - 0(4-0) + 3(-4-12)$$

$$= -36$$

$$(\vec{v}\times\vec{w})\cdot\vec{u} = \begin{vmatrix} 4 & -1 & 1 \\ 4 & 0 & 3 \\ 4 & 3 & 0 \end{vmatrix}$$

$$= 4\begin{vmatrix} -1 & 1 \\ 0 & 3 \end{vmatrix} - 3\begin{vmatrix} 4 & 1 \\ 4 & 3 \end{vmatrix} + 0\begin{vmatrix} 4 & -1 \\ 4 & 0 \end{vmatrix}$$

$$= 4(-3-0) - 3(12-4) + 0(0+4)$$

$$= -36$$

$$(\vec{u}\times\vec{w})\cdot\vec{v} = \begin{vmatrix} 4 & 0 & 3 \\ 4 & 3 & 0 \\ 4 & -1 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 0 & 3 \\ 3 & 0 \end{vmatrix} + 1\begin{vmatrix} 4 & 3 \\ 4 & 0 \end{vmatrix} + 1\begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix}$$

$$= 4(0-9) + 1(0-12) + 1(12-0)$$

$$= -36$$

$\therefore (\vec{u}\times\vec{v})\cdot\vec{w} = (\vec{v}\times\vec{w})\cdot\vec{u} = (\vec{u}\times\vec{w})\cdot\vec{v}$

Volume

$V = |-36|$
$= 36$ units cubed.

$V = |(\vec{u}\times\vec{v})\cdot w|$

Let $\vec{u} = -14i + 4j - 4k$, $\vec{v} = 2j - 7k$, $\vec{w} = 7i - 2j + 2k$

Which vectors are a) perpendicular b) parallel ?

Orthogonal vectors or perpendicular vectors
vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u} \cdot \vec{v} = 0$

$$\vec{u} \cdot \vec{v} = (-14 \cdot 0) + (4 \cdot 2) + (-4 \cdot -7) = 36$$

$\therefore$ $\vec{u}$ and $\vec{v}$ $\neq$ perpendicular.

$$\vec{u} \cdot \vec{w} = (-14 \cdot 7) + (4 \cdot -2) + (-4 \cdot 2) = -114$$

$\therefore$ $\vec{u}$ and $\vec{w}$ $\neq$ perpendicular

$$\vec{v} \cdot \vec{w} = (0 \cdot 7) + (2 \cdot -2) + (-7 \cdot 2) = -18$$

$\therefore$ $\vec{v}$ and $\vec{w}$ $\neq$ perpendicular

Parallel vectors
Non-zero vectors $\vec{u}$ and $\vec{v}$ are parallel if and only if $\vec{u} \times \vec{v} = 0$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ -14 & 4 & -4 \\ 0 & 2 & -7 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{vmatrix}$$

$$= i(-28+8) - j(98-0) + k(-28-0)$$
$$= -20i - 98j - 28k$$

$\vec{u} \times \vec{v}$ $\neq$ parallel

$$\vec{u} \times \vec{w} = \begin{vmatrix} i & j & k \\ -14 & 4 & -4 \\ 7 & -2 & 2 \end{vmatrix}$$

$$= i(8-8) - j(-28+28) + k(28-28)$$
$$= 0$$

$\vec{u} \times \vec{w}$ = parallel

$$\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 0 & 2 & -7 \\ 7 & -2 & 2 \end{vmatrix}$$

$$= i(4-14) - j(0+49) + k(0-14)$$
$$= -10i - 49j - 14k$$

$\vec{v} \times \vec{w}$ $\neq$ parallel let $\vec{u} = -9i + 3j - 3k$, $\vec{v} = j - 3k$, $\vec{w} = 3i - j + k$

Orthogonal vectors or perpendicular vectors
vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u}\cdot\vec{v} = 0$

$$\vec{u}\cdot\vec{v} = (-9\cdot 0) + (3\cdot 1) + (-3\cdot -3) \quad \therefore$$
$$= 12$$

$\vec{u}$ and $\vec{v}$ $\neq$ perpendicular.

$$\vec{u}\cdot\vec{w} = (-9\cdot 3) + (3\cdot -1) + (-3\cdot 1) \quad \therefore$$
$$= -33$$

$\vec{u}$ and $\vec{w}$ $\neq$ perpendicular

$$\vec{v}\cdot\vec{w} = (0\cdot 3) + (1\cdot -1) + (-3\cdot 1) \quad \therefore$$
$$= -4$$

$\vec{v}$ and $\vec{w}$ $\neq$ perpendicular

Parallel vectors
Non-zero vectors $\vec{u}$ and $\vec{v}$ are parallel if and only if $\vec{u}\times\vec{v} = 0$

$$\vec{u}\times\vec{v} = \begin{vmatrix} i & j & k \\ -9 & 3 & -3 \\ 0 & 1 & -3 \end{vmatrix} = i(-9+3) - j(27-0) + k(-9-0)$$
$$= -6i - 27j - 9k$$

$\vec{u}\times\vec{v}$ $\neq$ parallel

$$\vec{u}\times\vec{w} = \begin{vmatrix} i & j & k \\ -9 & 3 & -3 \\ 3 & -1 & 1 \end{vmatrix} = i(3-3) - j(-9+9) + k(9-9)$$
$$= 0$$

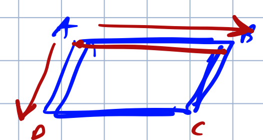
$\vec{u}\times\vec{w}$ = parallel

$$\vec{v}\times\vec{w} = \begin{vmatrix} i & j & k \\ 0 & 1 & -3 \\ 3 & -1 & 1 \end{vmatrix} = i(1-3) - j(0+9) + k(0-3)$$
$$= -2i - 9j + 3k$$

$\vec{v}\times\vec{w}$ $\neq$ parallel

Find the area of the parallelogram whose vertices are given below

$A(-1,4)$ $B(4,0)$ $C(8,3)$ $D(3,7)$



$$\vec{AB} = (4+1)i + (0-4)j = 5i - 4j$$
$$\vec{AD} = (3+1)i + (7-4)j = 4i + 3j$$

$$\vec{AB}\times\vec{AD} = \begin{vmatrix} i & j & k \\ 5 & -4 & 0 \\ 4 & 3 & 0 \end{vmatrix} = \cancel{i} + \cancel{j} + (15+16)k$$
$$= 31k$$

The area of the parallelogram ABCD is 31 square units.

Find the area of the triangle with $(-1,1,-2)$, $(-2,0,1)$ and $(0,-2,-1)$ as vertices.

Area of the triangle

$$\vec{AB} = (-2+1)i + (0-1)j + (1+2)k$$
$$= -i - j + 3k$$

$$\vec{AC} = (0+1)i + (-2-1)j + (-1+2)k$$
$$= i - 3j + k$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ 1 & -3 & 1 \end{vmatrix} = i(-1+9) - j(-1-3) + k(3+1)$$
$$= 8i + 4j + 4k$$

$$\rightarrow \frac{1}{2}\sqrt{8^2+4^2+4^2} = \frac{\sqrt{96}}{2} = \frac{\cancel{2}\sqrt{24}}{\cancel{2}} = \sqrt{24} \text{ or } 2\sqrt{6} \text{ Square units}$$

Area $= \frac{1}{2}|\vec{PQ} \times \vec{PR}|$

A line passes through the point $(-7,8,-7)$ and is parallel to the vector $9i + 3j + 7k$.

Find the standard parametric equations for the lines written using the components of the given vector and the coordinates of the given point.

let $z = -7 + 7t$

$P_0 = (x_0, y_0, z_0)$ equal to $(-7, 8, -7)$

$V_1 i + V_2 j + V_3 k$ equal to $9i + 3j + 7k$

so, $x = -7 + 9t$ $\quad y = 8 + 3t$ $\quad z = -7 + 7t$

A line passes through the points $P(5,10,2)$ and $Q(-7,-10,-1)$. Find the standard parametric equations for the lines written using the base point $P(5,10,2)$ and the components of the vector $\vec{PQ}$

let $z = 2 - 3t$

$P(5,10,2)$ $\quad Q(-7,-10,-1)$

the vector is: $\vec{PQ} = (-7-5)i + (-10-10)j + (-1-2)k$
$= -12i - 20j - 3k$

the vector is parallel to the line with $P(5,10,2)$ thus,

$x = 5 - 12t$ $\quad y = 10 - 20t$ $\quad z = 2 - 3t$

Find parametric equations for the line through the origin parallel to the vector $3j + 8k$.

Let $z = 8t$.

the line throug the origin parallel to the vector is: $3j + 8k$

$\rightarrow$ $x = 0$ $\quad y = 3t$ $\quad z = 8t$, $\quad -\infty < t < \infty$

the origin is located at $(0,0,0)$

Find parametric equations for the line $(-2,1,7)$ parallel to the z-axis.

Let $z = 7 + t$.

vector along z-axis is : $0i + 0j + zk$

Unit vector : $v = \dfrac{0}{\sqrt{0^2+0^2+z^2}}i + \dfrac{0}{\sqrt{0^2+0^2+z^2}}j + \dfrac{z}{\sqrt{0^2+0^2+z^2}}k$

$= \frac{0}{z}i + \frac{0}{z}j + \frac{z}{z}k$

the parametric equation of the line throug $(-2,1,7)$ and parallel to $0i + 0j + zk$ is :

$x = -2 + 0t$ $\quad y = 1 + 0t$ $\quad z = 7 + t$

$= -2$ $\quad = 1$

Find parametric equations for the line $(2,6,-6)$ perpendicular to the plane $7x + 5y + 5z = 24$.

$\perp$ = perpendicular

Let $z = -6 + 5t$

the normal vector $7i + 5j + 5k$ is $\perp$ to the plane $7x + 5y + 5z = 24$.

the required line is $\perp$ to plane $7x + 5y + 5z = 24$.

$P_0(x_0, y_0, z_0) = (2,6,-6)$ and $v_1 i + v_2 j + v_3 k = 7i + 5j + 5k$

$\therefore$ $x = 2 + 7t$ $\quad y = 6 + 5t$ $\quad z = -6 + 5t$

Find parametric equations for the line shown below x-axis.

Let $x = t$.

the x-axis $= i + 0j + 0k$

the point is $= (0,0,0)$

the line is : $x = t$ $\quad y = 0$ $\quad z = 0$

Find a parametrization for the line segment joining the points $P(0,0,0)$ and $Q(10, 2, \frac{5}{7})$.

Draw coordinate axes and sketch the segment, indicating the direction of increasing t for parametrization.

① Find the vector equation for line segment $P(0,0,0)$ to $Q(10, 2, \frac{5}{7})$.

$r = r_0 + t(r_1 - r_0) \quad 0 \leq t \leq 1.$

$r_0 = \langle 0,0,0 \rangle \qquad r_1 = \langle 10, 2, \frac{5}{7} \rangle$

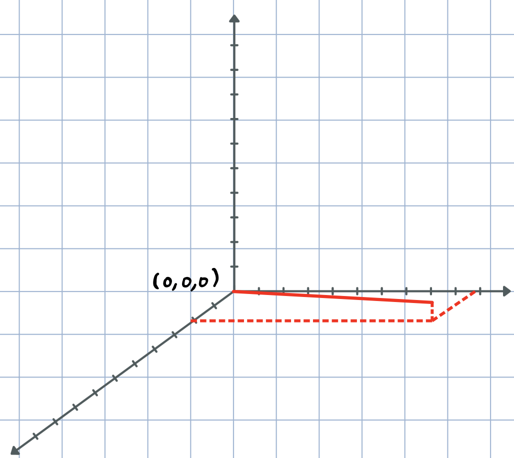
$r = \langle 0,0,0 \rangle + t \ \langle 10, 2, \frac{5}{7} \rangle - \langle 0,0,0 \rangle \ , \quad 0 \leq t \leq 1$

$= \langle 0,0,0 \rangle + t \ \langle 10-0, 2-0, \frac{5}{7}-0 \rangle, \quad 0 \leq t \leq 1$

$= \langle 0,0,0 \rangle + t \ \langle 10, 2, \frac{5}{7} \rangle, \quad 0 \leq t \leq 1$

$= \langle 0,0,0 \rangle + \langle 10t, 2t, \frac{5}{7}t \rangle, \quad 0 \leq t \leq 1$

$r = \langle 10t, 2t, \frac{5}{7}t \rangle, \quad 0 \leq t \leq 1$



$r = \langle x, y, z \rangle$

$r = \langle 10t, 2t, \frac{5}{7}t \rangle$

$\langle x, y, z \rangle = \langle 10t, 2t, \frac{5}{7}t \rangle$

Parametric equation for the line segment $(0,0,0)$ and $(10, 2, \frac{5}{7})$.

$= \ x = 10t \quad y = 2t \quad z = \frac{5}{7}t$

Find a parametrization for the line segment joining the points $P(0,7,5)$ and $Q(0,-7,5)$.

Draw coordinate axes and sketch the segment, indicating the direction of increasing t for parametrization.

① Find the vector equation for line segment $P(0,7,5)$ to $Q(0,-7,5)$.

$$r = r_0 + t(r_1 - r_0), \quad 0 \le t \le 1.$$

$$r_0 = \langle 0,7,5 \rangle$$
$$r_1 = \langle 0,-7,5 \rangle$$

$$\therefore \quad r = \langle 0,7,5 \rangle + t(\langle 0,-7,5 \rangle - \langle 0,7,5 \rangle)$$
$$r = \langle 0,7,5 \rangle + t(\langle 0-0, -7-7, 5-5 \rangle)$$
$$r = \langle 0,7,5 \rangle + t(0, -14, 0)$$
$$r = \langle 0,7,5 \rangle + t(0), -14t, +(0)$$
$$r = \langle 0, 7-14t, 5 \rangle$$

$$r = \langle x,y,z \rangle$$

$$r = \langle 0t + 7-14t, 5t \rangle$$

$$\therefore \quad \langle x,y,z \rangle = \langle 0t + 7-14t + 5t \rangle$$

$t=0$

$x=0$

$y=7$

$z=5$

For $P_P \to t = 0$ $x=$ $x=$

$y=$ $y=$

For $P_Q \to t = 1$ $z=$ $z=$

$x = -14$

$y = 7 - 14$

$z = 5 - 14$

$$r = r_0 + t(r_1 - r_0) \quad 0 \le t \le 1.$$

$r_0 = \langle 0,0,0 \rangle \quad r_1 = \langle 10, 2, \frac{5}{7} \rangle$

$r = \langle 0,0,0 \rangle + t \langle 10, 2, \frac{5}{7} \rangle - \langle 0,0,0 \rangle, \quad 0 \le t \le 1$

$= \langle 0,0,0 \rangle + t \langle 10-0, 2-0, \frac{5}{7}-0 \rangle, \quad 0 \le t \le 1$

$= \langle 0,0,0 \rangle + t \langle 10, 2, \frac{5}{7} \rangle, \quad 0 \le t \le 1$

$= \langle 0,0,0 \rangle + \langle 10t, 2t, \frac{5}{7}t \rangle, \quad 0 \le t \le 1$

$r = \langle 10t, 2t, \frac{5}{7}t \rangle, \quad 0 \le t \le 1$

$r = \langle x,y,z \rangle$

$r = \langle 10t, 2t, \frac{5}{7}t \rangle$

$\langle x,y,z \rangle = \langle 10t, 2t, \frac{5}{7}t \rangle$

Parametric equation for the line segment $(0,0,0)$ and $(10, 2, \frac{5}{7})$.

$= \quad x = 10t \quad y = 2t \quad z = \frac{5}{7}t$

$x = 0$

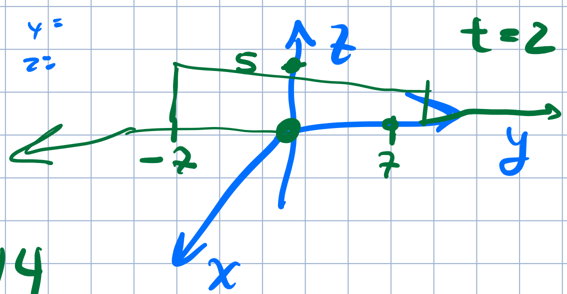
$y = 7 - 14t$

$z = 5$

$t = 0 \to y = 7$

$t = 1 \to y = -7$

$t = 2$



Find a parametrization for the line segment joining the points $P(6,0,6)$ and $Q(0,6,0)$.

Draw coordinate axes and sketch the segment, indicating the direction of increasing t for parametrization.

$P(6,0,6) \quad Q(0,6,0)$

$$\vec{PQ} = (0-6)i + (6-0)j + (0-6)k$$
$$= -6i + 6j - 6k$$

$P_0(6,0,6)$ // to $\vec{PQ} = -6i + 6j - 6k$

$x = 6 + t(-6) \quad y = 0 + t(6) \quad z = 6 + t(-6)$

$x = 6 - 6t \quad y = 6t \quad z = 6 - 6t$

at $t = 0$ the line passes through $P(6,0,6)$

at $t = 1$ the line passes through $Q(0,6,0)$

therefore, $x = 6 - 6t,\ y = 6t,\ z = 6 - 6t, \quad 0 \le t \le 1$

Find the equation for the plane through the point $P_0(6,2,5)$ and normal to the vector $n = 2i + 9j + 2k$

since n touches a plane at $P_0(6,2,5)$

$D = 2(6) + 9(2) + 2(5)$

$D = 40$

substitute: $2x + 9y + 2z = 40$

Find the equation for the plane through the point $P_0(2,4,4)$, $Q_0(-3,4,3)$ and $R_0(2,-2,1)$

$$\vec{P_0Q_0} = (-3-2)i + (4-4)j + (3-4)k$$
$$= -5i - k$$

$$\vec{P_0R_0} = (2-2)i + (-2-4)j + (1-4)k$$
$$= -6j - 3k$$

$$\vec{P_0Q_0} \times \vec{P_0R_0} = \begin{vmatrix} i & j & k \\ -5 & 0 & -1 \\ 0 & -6 & -3 \end{vmatrix}$$

$$= i(0-6) - j(15-0) + k(30-0)$$
$$\vec{P_0Q_0} \times \vec{P_0R_0} = -6i - 15j + 30k$$

Vector $\vec{P_0Q_0} \times \vec{P_0R_0}$ passing through $P_0(2,4,4)$

$$D = -6x - 15y + 30z$$
$$= -6(2) - 15(4) + 30(4)$$
$$= 48$$

$$\therefore -6x - 15y + 30z = 48$$

$$= \frac{3(-2x - 5y + 10z)}{3} = \frac{48}{3}$$

$$= -2x - 5y + 10z = 16$$

Find the equation for the plane through the point $P_0(-3,3,-3)$, $Q_0(5,5,3)$ and $R_0(4,1,2)$

$\overrightarrow{P_0Q_0} = (5+3)i + (5-3)j + (3+3)k$
$= 8i + 2j + 6k$

$\overrightarrow{P_0R_0} = (4+3)i + (1-3)j + (2+3)k$
$= 7i - 2j + 5k$

$\overrightarrow{P_0Q_0} \times \overrightarrow{P_0R_0} = \begin{vmatrix} i & j & k \\ 8 & 2 & 6 \\ 7 & -2 & 5 \end{vmatrix}$

$= i(10+12) - j(40-42) + (-16-14)k$
$= 22i + 2j - 30k$

$D = 22x + 2y - 30z$
$= 22(-3) + 2(3) - 30(-3)$
$= 30$

$\therefore \quad 22x + 2y - 30z = 30$

$\dfrac{2(11x + y - 15z)}{2} = \dfrac{30}{2}$

$11x + y - 15z = 15$

Find the equation for the plane through $P_0(1,-6,-2)$ $\perp$ to the line $x = 1+t$, $y = -6-4t$, $z = -2t$, $-\infty \le t \le \infty$

$n = i - 4j - 2k$ then $A = 1$ $B = -4$ $C = -2$

$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

$1(x-1) - 4(y+6) - 2(z+2)$

$x - 1 - 4y - 24 - 2z - 4 = 0$

$x - 4y - 2z = 1 + 24 + 4$

$D = x - 4y - 2z$
$D = 1 + 24 + 4$
$= 29$

$x - 4y - 2z = 29$

Find the distance from the point to the line $P(-4,8,1)$; $x = -4+9t$, $y = 8+5t$, $z = 1$.

$S(-4,8,1)$ and line $x = -4+9t$, $y = 8+5t$, $z = 1$

Now, $\overrightarrow{PS} = (-4+4)i + (8-8)j + (1-1)k = 0i + 0j + k$

Vector $\vec{v}$ along the given line is : $\vec{v} = 9i + 5j + 0k$

Now, $\overrightarrow{PS} \times \vec{v} = 0$

Find the distance from the point to the line

$(-1,-4,-2)$ $x=-3-2t$, $y=-2+4t$, $z=4+t$

the point to the line $x=-3-2t$, $y=-2+4t$, $z=4+t$ is $P(-3,-2,4)$.

evaluate $\vec{PS}$: $(-1+3)i + (-4+2)j + (-2-4)k$

$= 2j - 2j - 6k$

$\vec{v} = -2i + 4j + k$

Find $|\vec{PS} \times \vec{v}|$: $\vec{PS} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & -2 & -6 \\ -2 & 4 & 1 \end{vmatrix}$

$= i(-2+24) - j(2-12) + k(8-4)$

$= 22i + 10j + 4k$

$|\vec{PS} \times \vec{v}| = \sqrt{22^2 + 10^2 + 4^2}$

$= \sqrt{600} = 10\sqrt{6}$

$|\vec{v}| = \sqrt{(-2)^2 + 4^2 + 1^2}$

$= \sqrt{21}$

distance $= \dfrac{10\sqrt{6}}{\sqrt{21}}$

$d = \dfrac{|\vec{PS} \times \vec{v}|}{|\vec{v}|}$

$= \dfrac{10\sqrt{126}}{21}$

Find the distance d, between the point $S(5,8,6)$ and the plane $5x + 7y + 1z = 40$.

$\vec{n} = 5i + 7j + k$

to find $\vec{PS}$ use x-intercept.
Let $y=0$ $z=0$.

$40 = 5x + 7y + 1z$
$40 = 5x$
$x = 8$ thus, $(8,0,0)$ is a point on the plane.

$\vec{PS} = (5-8)i + (8-0)j + (6-0)k$
$= -3i + 8j + 6k$

$d = \left| \dfrac{-3(5) + 8(7) + 6(1)}{\sqrt{5^2 + 7^2 + 1^2}} \right|$ $d = \left| \vec{PS} \cdot \dfrac{\vec{n}}{|\vec{n}|} \right|$

Find the distance d, between the point $S(4,3,1)$ and the plane $1x + 7y + 7z = 6$.

$\vec{n} = i + 7j + 7k$

to find $\vec{PS}$ use x-intercept.
Let $y=0$ $z=0$.

$6 = 1x + 7y + 7z$
$6 = 1x$
$x = 6$ thus, $(6,0,0)$ is a point on the plane.

$\vec{PS} = (4-6)i + (3-0)j + (1-0)k$
$= -2i + 3j + 1k$

$d = \left| \dfrac{-2(1) + 3(7) + 1(7)}{\sqrt{1^2 + 7^2 + 7^2}} \right|$ $d = \left| \vec{PS} \cdot \dfrac{\vec{n}}{|\vec{n}|} \right|$

$= \left| \frac{47}{\sqrt{75}} \right| \approx 5.43$

$= \left| \frac{26}{\sqrt{99}} \right| \approx 2.61$

Find the angle between the planes
$6x + 5y = -11$ and

① $6x + 5y = -11$
② $3x + 6y + 2z = -8$

the vectors normal planes ① and ② are:

$\vec{n}_1 = 6i + 5j$
$\vec{n}_2 = 3i + 6j + 2k$

the angle between the planes is defined by the angle between their normals

$\therefore \quad \theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|}\right)$

$= \cos^{-1}\left(\frac{(6i + 5j) \cdot (3i + 6j + 2k)}{\sqrt{6^2 + 5^2} \cdot \sqrt{3^2 + 6^2 + 2^2}}\right)$

$= \cos^{-1}\left(\frac{18 + 30 + 0}{\sqrt{61}\sqrt{49}}\right)$

$= \cos^{-1}\left(\frac{48}{7.810 \cdot 7}\right) \approx 0.499$

Find the angle between the planes
$3x + 3y = -5$ and $9x + 5y + 5z = 15$

① $3x + 3y = -5$
② $9x + 5y + 5z = 15$

the vectors normal planes ① and ② are:

$\vec{n}_1 = 3i + 3j$
$\vec{n}_2 = 9i + 5j + 5k$

the angle between the planes is defined by the angle between their normals

$\therefore \quad \theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|}\right)$

$= \cos^{-1}\left(\frac{(3i + 3j) \cdot (9i + 5j + 5k)}{\sqrt{3^2 + 3^2} \cdot \sqrt{9^2 + 5^2 + 5^2}}\right)$

$= \cos^{-1}\left(\frac{27 + 15}{\sqrt{18}\sqrt{131}}\right)$

$= \cos^{-1}\left(\frac{42}{4.243 \cdot 11.455}\right) \approx 0.527$

Find the point, P, at which the line intersects the plane.
$x = 10 - 3t, \quad y = -6 + 9t, \quad z = -10 + 5t; \quad -3x + 2y + 8z = 16.$

L: $x = 10 - 3t, \quad y = -6 + 9t, \quad z = -10 + 5t;$
P: $-3x + 2y + 8z = 16.$

$-3(10 - 3t) + 2(-6 + 9t) + 8(-10 + 5t) = 16$
$= -30 + 9t - 12 + 18t - 80 + 40t = 16$
$= 9t + 18t + 40t = 16 + 30 + 12 + 80$
$67t = 138$
$t = \frac{138}{67}$

$x = 10 - 3\left(\frac{138}{67}\right) \qquad y = -6 + 9\left(\frac{138}{67}\right) \qquad z = -10 + 5\left(\frac{138}{67}\right)$

$\frac{256}{67}, \frac{840}{67}, \frac{20}{67}$

Oscar Garcia | 09/03/

Homework: **hw 2** Question 38, 12.5.63 HW Score: 92.08%, 36.83 of 40 points

Points: 0 of 1

Find a parametrization of the line in which the planes $x+2y-2z=3$ and $3x+3y-3z=3$ intersect.

Find the parametrization of the line. Use a point with $z=0$ on the line to determine the parametrization.

$x=\square$, $y=\square$, $z=\square$, $-\infty<t<\infty$

$$x+2y-2z=3 \quad --(1)$$

$$3x+3y-3z=3 \quad --(2)$$

$$\bar{n}_1 = 1\hat{i}+2\hat{j}-2\hat{k}$$

$$\bar{n}_2 = 3\hat{i}+3\hat{j}-3\hat{k}$$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 3 & -3 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & -2 \\ 3 & -3 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & -2 \\ 3 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix}$$

$$\bar{n}_1 \times \bar{n}_2 = \left[\overset{-6}{\cancel{(2)(-3)}} - \overset{-6}{\cancel{(-2)(3)}}\right]\hat{i} - \left[\overset{-3}{\cancel{(1)(-3)}} - \overset{-6}{\cancel{(-2)(3)}}\right]\hat{j} + \left[\overset{3}{\cancel{(1)(3)}} - \overset{6}{\cancel{(3)(2)}}\right]\hat{k}$$

$$= (-6+6)\hat{i} - (-3+6)\hat{j} + (3-6)\hat{k}$$

$$= 0\hat{i} - 3\hat{j} - 3\hat{k}$$

* To find a point on the line, we substitute $z=0$ into plane equations & solve:

$$x+2y-\overset{0}{\cancel{2z}}=3 \quad --(1) \Rightarrow x+2y=3 \Rightarrow x=3-2y \Rightarrow x=3-2(2)$$

$$x = 3-4$$

$$\underline{\underline{x=-1}}$$

$$3x+3y-\overset{0}{\cancel{3z}}=3 \quad --(2) \Rightarrow 3x+3y=3$$

$$3(3-2y)+3y=3$$

$$9-6y+3y=3$$

$$6=3y$$

$$6/3 = \underline{\underline{2=y}}$$

$\therefore$ the point is $P(-1,2,0)$

* $\vec{V} = \bar{n}_1 \times \bar{n}_2 = 0\hat{i}-3\hat{j}-3\hat{k}$

$$x=-1+0t = -1$$

$$y=2-3t$$

$$z=0-3t = -3t$$

$$\boxed{\begin{aligned} x &= -1 \\ y &= 2-3t \\ z &= -3t \end{aligned}}$$

Homework: hw 2 — Question 39, 12.5.67 — Part 1 of 2 — HW Score: 94.58%, 37.83 of 40 points — Points: 0 of 1 — Save

Use the standard parametrization of a line through a point parallel to a vector to generate a parametrization of the line through $P_1(6,-4,1)$ parallel to $v_1 = 6i - j + k$. Then generate another parametrization of the line using the point $P_2(-6,-2,-1)$ and the vector $v_2 = -3i + \frac{1}{2}j - \frac{1}{2}k$.

Find the parametrization using P_1 and v_1.

$x = \square$, $y = \square$, $z = \square$, $-\infty < t < \infty$

(Type expressions using t as the variable.)

$P_1(6,-4,1)$ (x_0, y_0, z_0) $\quad \vec{V_1} = 6\hat{i} - \hat{j} + \hat{k}$ (V_1, V_2, V_3) $\Rightarrow$ 1st Set

$P_2(-6,-2,-1)$ (x_0, y_0, z_0) $\quad \vec{V_2} = -3\hat{i} + 0.5\hat{j} - 0.5\hat{k}$ (V_1, V_2, V_3) $\Rightarrow$ 2nd Set

* The parametric eqt. of a line thru $P(x_0, y_0, z_0)$ & // to $\vec{V} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ is given by:

$$x = x_0 + t v_1$$
$$y = y_0 + t v_2$$
$$z = z_0 + t v_3$$

$\Rightarrow$ 1st Set

$x = 6 + 6t$, $y = -4 - 1t$, $z = 1 + 1t$ $\therefore$ $x = 6 + 6t$, $y = -4 - t$, $z = 1 + t$

$\Rightarrow$ 2nd Set

$x = -6 - 3s$, $y = -2 + 0.5s$, $z = -1 - 0.5s$ $\therefore$ $x = -6 - 3s$, $y = -2 + \frac{1}{2}s$, $z = -1 - \frac{1}{2}s$

Homework: hw 2 — Question 40, 12.5.71 — HW Score: 97.08%, 38.83 of 40 points — Points: 0 of 1 — Save

Is the line $x = -1 - 2t$, $y = -1 + 6t$, $z = -2t$ parallel to the plane $2x + y + z = 9$? Give reasons for your answer.

Since the [▼] product of the vector parallel to the given line and the normal vector of the given plane is $\square$, the line [▼] parallel to the plane.

Given that:

Line: $x = -1 - 2t$, $y = -1 + 6t$, $z = -2t$ --- (1) (v_1, v_2, v_3)

Plane: $2x + y + z = 9$ -- (2) (n_1, n_2, n_3)

The vector $\perp$ to plane (2) is $\vec{n} = 2\hat{i} + 1\hat{j} + 1\hat{k}$
& the vector // to line is $\vec{V} = -2\hat{i} + 6\hat{j} - 2\hat{k}$

Now, $\vec{n} \cdot \vec{V} = 2(-2) + (1)(6) + (1)(-2)$
$-4 + 6 - 2 = 0$

$\therefore$ It's parallel ✓

Find a parametrization for the line segment joining the points $P(7,0,7)$ and $Q(0,7,0)$.

Draw coordinate axes and sketch the segment, indicating the direction of increasing t for parametrization.

$P(7,0,7) \quad Q(0,7,0)$

$$\vec{PQ} = (0-7)i + (7-0)j + (0-7)k$$
$$= -7i + 7j - 7k$$

$P_0(7,0,7)$ // to $\vec{PQ} = -7i + 7j - 7k$

$x = 7 + t(-7) \quad y = 0 + t(7) \quad z = 7 + t(-7)$

$x = 7 - 7t \quad y = 7t \quad z = 7 - 7t$

at $t = 0$ the line passes through $P(7,0,7)$

at $t = 1$ the line passes through $Q(0,7,0)$

therefore, $x = 7 - 7t$, $y = 7t$, $z = 7 - 7t$, $0 \le t \le 1$

$t = 0$
$x = 7$
$y = 0$
$z = 7$

$t = 1$
$x = 0$
$y = 7$
$z = 0$

$t = 2$
$x = -7$
$y = 14$
$z = -7$